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New kind of phase separation in a CA traffic model with anticipation

M E Lárraga[1], **J A del Río**[1] **and A Schadschneider**[2]

[1] Centro de Investigación en Energía, UNAM, AP 34, 62580 Temixco, Mor., Mexico
[2] Institute for Theoretical Physics, Universität zu Köln, 50937 Köln, Germany

E-mail: mel@cie.unam.mx, antonio@servidor.unam.mx and as@thp.uni-koeln.de

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Abstract

A cellular automaton model of traffic flow taking into account velocity anticipation is introduced. The strength of anticipation can be varied to describe different driving schemes. We find a new phase separation into a free-flow regime and a so-called v-platoon in an intermediate density regime. In a v-platoon all cars move with velocity v and have vanishing headway. The velocity v of a platoon only depends on the strength of anticipation. At high densities, a congested state characterized by the coexistence of a 0-platoon with several v-platoons is reached. The results are not only relevant for automated highway systems, but also help to elucidate the effects of anticipation that play an essential role in realistic traffic models. From a physics point of view the model is interesting because it exhibits phase separation with a condensed phase in which particles move coherently with finite velocity coexisting with either a non-condensed (free-flow) phase or another condensed phase that is non-moving.

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1. Introduction

In the last few years, the continuous increase of traffic demand has prompted authorities around the world to place more emphasis on improving the efficiency and capacity of the roadway systems. Ecological considerations, space and budgetary constraints have limited solving traffic congestion by upgrading and constructing new roadway systems. Advanced technologies for vehicular traffic have been developed as a means to improve the management of the existing system and thus to solve traffic congestion, environmental issues and improve traffic safety. However, to achieve these aims, an accurate forecast of the impact of these technologies is critical before their final deployment.

Testing these advanced technologies on real traffic is not always feasible. In contrast, computer simulations as a means of evaluating control and management strategies in traffic systems have gained considerable importance because of the possibility of taking into account the dynamical aspects of traffic (see, for example, [1–3]) and assessing the performance of a given advanced technology in a short time.

Cellular automata (CA) models for traffic flow [4, 5] have shown the ability to capture the basic phenomena in traffic flow [6]. Cellular automata are dynamic models in which space, time and state variables are discrete. Discrete space consists of a regular grid of cells, where each one can be in one of a finite number k of possible states. All cells are updated in discrete time-steps. The new state of a cell is determined by the actual state of the cell itself and its neighbour cells. This local interaction allows the capture micro-level dynamics and propagates it to macro-level behaviour. The discrete nature of CA makes it possible to simulate large realistic traffic networks using a microscopic model faster than real time [5, 7, 8]. Nowadays, several theoretical studies and practical applications have improved the understanding of empirical traffic phenomena (see, e.g., [6, 9–11]). Moreover, CA models have proved to be a realistic description of vehicular traffic in dense networks [7, 8].

In this paper, we propose a single-lane probabilistic model based on the first CA model of Nagel and Schreckenberg (NaSch model) [4] to describe the effects of several anticipation schemes in traffic flow. Here anticipation means that drivers estimate their leader's velocities for future time-steps [12–18]. This can lead to an increase of the vehicular capacity and a decrease of the speed variance. However, incorporating different driving strategies requires a safety distance with respect to the preceding vehicles. For this purpose we introduce a new parameter in the deceleration process, called the *anticipatory driving parameter*, to estimate the velocity of the preceding vehicle. This estimation, plus the real spatial distance to the leading vehicle, establishes a safe distance among vehicles. By appropriately tuning this new parameter different traffic situations of non-automated, automated and mixed traffic can be considered. Furthermore, the anticipatory driving parameter is relevant for automated highway systems (AHS) [19, 20].

According to simulation results from our proposed model, the parameters can be adjusted to reproduce empirical fundamental diagrams (flow versus density curves) of real non-automated traffic. In addition, simulation results in the case of high anticipation (like automation) describe one of the interesting phenomena in traffic flow, *formation of platoons*. We will show that, in contrast to models without anticipation, *dense* platoons can be formed where all cars move coherently with some finite velocity $v > 0$. The mechanism for platoon formation is not only of great importance for AHS to increase highway capacity in a much safer way [21], but also helps to understand the essential role of anticipation effects in realistic traffic models. By varying the anticipatory driving parameter three different regimes, characterized by different slopes of the fundamental diagrams, can be observed. Apart from a free-flow and a congested phase, an additional regime where platoons of cars move with the same velocity $v < v_{\max} - 1$ exists.

The paper is organized as follows. In section 2 we define a modified NaSch model to consider different driving strategies. It contains a new parameter to establish a velocity-dependent safe distance. In section 3, we present the results of our investigations. We show results for the fundamental diagram and different values of the anticipation parameter. Phase separation into a free-flow regime and so-called v-platoon is observed in a certain intermediate density regime. For large densities, in the congested state phase separation into a dense jam (0-platoon) and v-platoons is observed. This is also interesting in relation to recent general studies about phase separation in one-dimensional nonequilibrium systems [22, 23]. The flow structure determined by the existence of dense platoons with velocity v is calculated.

Analytical results are in excellent agreement with results from computer simulations. In the concluding section 4 we summarize our results and discuss the relevance of our results for traffic models, real traffic and the physics of phase separation in driven diffusive systems.

2. Definition of the model

The proposed model is defined on a one-dimensional lattice of L cells with periodic boundary conditions, which corresponds to a ring topology with the number of vehicles preserved. Each cell is either empty, or it is occupied by just one vehicle travelling with a discrete velocity v at a given instant of time. All vehicles have a velocity $v \in \{0, \ldots, v_{\max}\}$. For simplicity only one type of vehicle is considered here. The time-step (Δt) is taken to be 1 s.

Let v_i and x_i denote the current velocity and position, respectively, of vehicle i, and v_p and x_p be the velocity and position, respectively, of the vehicle ahead (preceding vehicle) at a fixed time; $d_i := x_p - x_i - 1$ denotes the distance (or headway, i.e. number of empty cells) in front of the vehicle in position x_i.

The dynamics of the model is defined by the following set of rules, that are applied to all N vehicles on the lattice in each time-step:

R1: Acceleration. If $v_i < v_{\max}$, the velocity of the car i is increased by 1, i.e.,

$$v_i \rightarrow \min(v_i + 1, v_{\max}).$$

R2: Randomization. If $v_i > 0$, the velocity of the car i is decreased randomly by one unit with probability R, i.e.,

$$v_i \rightarrow \max(v_i - 1, 0) \quad \text{with probability } R.$$

R3: Deceleration. If $d_i^s < v_i$, where (with a parameter $0 \leqslant \alpha \leqslant 1$)

$$d_i^s = d_i + \left[(1-\alpha) \cdot v_p + \tfrac{1}{2}\right]$$

the velocity of the car i is reduced to d_i^s. $[x]$ denotes the integer part of x, i.e. $\left[x + \frac{1}{2}\right]$ corresponds to rounding x to the next integer value. The new velocity of the vehicle i is therefore

$$v_i \rightarrow \min\left(v_i, d_i^s\right).$$

R4: Vehicle movement. Each car is moving forward according to its new velocity determined in steps 1–3:

$$x_i \rightarrow x_i + v_i.$$

Thus state updating is divided into two stages, first velocity (rules $R1$, $R2$ and $R3$), second position (rule $R4$). Note that this division follows the scheme in differential equation integration that first updates the time derivative and then the value of the state. It is important to mention that we are changing the order of the rules in comparison with the NaSch model since $R2$ is applied before $R3$. The reason for this change is that with $R2$ being applied after $R3$, as in the original NaSch model, cars are unable to adjust to the randomization-reduced velocities of the traffic in front. Therefore the model would no longer be automatically collision free, as is the NaSch model and most other CA traffic models. Our choice reflects our wish to model the behaviour of anticipatory drivers rather than reactive drivers.

Rule $R1$ indicates that all the drivers would like to reach the maximum velocity when possible. Rule $R2$ takes into account the different behavioural patterns of the individual drivers in which with no apparent reason a driver decreases its speed. These situations include, for example, cases of overreaction in braking or incidents along the highway that distract drivers, and random fluctuations.

Rule $R3$ is the main modification to the original NaSch model [4]. In this rule the distances between the ith and $(i+1)$th vehicles, and their corresponding velocities are considered. Knowledge of the preceding vehicle's velocity is incorporated through the *anticipatory driving parameter* α with range $0 \leqslant \alpha \leqslant 1$. Note that, by only varying the parameter α in the term $d_i^s = d_i + [(1-\alpha)v_p + 1/2]$, different driving strategies can be modelled. If α takes its maximum value ($\alpha = 1$), the speed of the vehicle ahead is not considered in the deceleration process (no anticipation). In contrast, when $\alpha = 0$ the speed of the vehicle ahead is considered without restrictions [20]. This last case is equivalent to taking into account the distance between two cars and the exact velocity of the vehicle ahead, i.e., to allow that a vehicle may be behind the other one with the same speed without the existence of an additional safe distance between them (only the distance included in the cell size). The case $\alpha = 0$ occurs with either a very aggressive driver or when vehicles can obtain information about the velocity of vehicles ahead[3] to allow small distances between vehicles (e.g. of the order 1 m). Intermediate values for α thus represent different safe spacing policies or strengths of anticipation in the vehicles. Platooning schemes [21] imply values of α closer to zero and demand additional requirements to preserve safety, such as coordinated braking [19]. Independent vehicle driving with low strengths of anticipation implies values of α closer to 1 in order to preserve safety levels. Therefore, the value of the parameter α will be established according to the desired policy of the safe distance among vehicles.

There is a price to pay with this modification that limits deceleration values. It could be that deceleration of a vehicle also implies decelerations of the following vehicles. Step $R3$ which assures that collisions are avoided, is then applied sequentially to take into account the limited deceleration capability. The final configuration is independent of the starting point of this sequential updating. In order to determine v_i consistently for all cars, $R3$ should be iterated at most $v_{\max}$ times in systems with periodic boundary conditions. For example, the most critical scenario is a chain of vehicles, each one of them travelling at maximum speed, with the first vehicle running into a stopped vehicle. In order to set all velocities of vehicles in the chain consistently within the maximally allowable deceleration, $R3$ has to be iterated $v_{\max}$ times, assuming that the velocity decreases by one from one vehicle to another in the chain.

In real situations, the drivers always estimate the velocity of the preceding vehicle and according to this and their driving style (relaxed or aggressive) they choose a safe headway distance. Variation of α also allows us to model these aspects. Thus, the proposed model is able to represent different anticipatory driving strategies, and model the safe distance required with only one parameter α.

We emphasize that the CA model as presented here is a minimal model in the sense that all four steps $R1$–$R4$ are necessary to reproduce the basic features of real traffic, however, additional rules may be needed to capture more complex situations [13].

3. Simulation results

To simulate the CA model proposed in the previous section, the typical length of a cell is around 7.5 m. It is interpreted as the length of a vehicle plus the distance between cars in a dense jam, but it can be suitably adjusted according to the problem under consideration. With this value of the cell size and a time-step of 1 s, $v = 1$ corresponds to moving from one cell to the downstream neighbour cell in one time-step, and translates to 27 km h^{-1} in real units. The maximum velocity is set to $v_{\max} = 5$, equivalent to 135 km h^{-1}. The total number of cells is assumed to be $L = 10^4$, and the density ρ is defined as $\rho = N/L$, where N is the number of

[3] This will happen in automated highway systems or vehicles equipped with appropriate sensors [24].

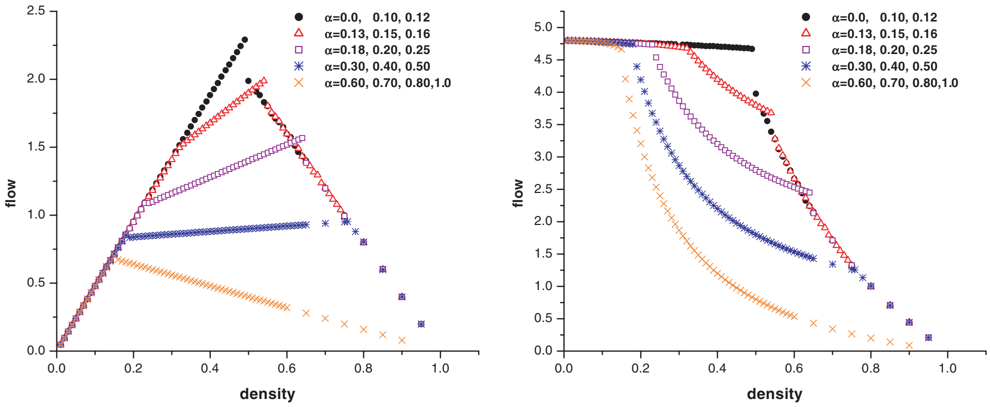

Figure 1. Left: fundamental diagram for different values of the anticipation parameter α. The legend indicates the effect of rounding in the estimation of the velocity of the preceding car, defined in rule $R3$. Right: relationship between mean velocity and density for $R = 0.2$ and different values of α.

cars on the highway. Initially, N vehicles are distributed randomly on the lane around the loop with an initial speed taking a discrete random value between 0 and $v_{\max}$. Since the system is closed, the density remains constant with time.

All the simulation data presented in this work have been generated by simulations of $L = 10^4$ and $T = 15L$ time-steps. In order to analyse the results, the first $10L$ time-steps of the simulation are discarded to let transients die out and the system reach its steady state. Then the simulation data are averaged over the final $5L$ time-steps.

Velocities are updated according to the velocity updating rules $R1$–$R2$–$R3$ and then all cars are moved forward in step $R4$. For each simulation a value for the parameter α is established by taking into account the desired strength of anticipation and thus, controlling the safe distance among vehicles. In the following, the value of α is the same for all vehicles (homogeneous drivers).

The fundamental diagram characterizes the dependence of the vehicle flow on density and is one of the most important criteria to show that the model reproduces traffic flow behaviour. Comparing with empirical data we find a good agreement by choosing the model parameters $R = 0.2$ and $\alpha = 0.75$. This α value corresponds to cautious estimation of the preceding car's velocity.

Variation of α makes it possible to consider several anticipation strategies, e.g. non-automated, mixed and automated traffic flow, and so go beyond previous analyses.

3.1. Modelling different anticipation schemes

Determination of the impact of different driving strategies is important in order to propose automated traffic alternatives. Following that proposal, we decided to investigate the traffic flow behaviour using our model. As mentioned above, the parameter α represents the way in which different driving strategies adopt a safe spacing policy or strength of anticipation in the vehicles. By varying the parameter α, these strategies can be tuned. In figure 1 we show the fundamental diagram of the proposed model with a fixed value of $R = 0.2$ and different values of α. Each curve includes multiple values of α as a consequence of the rounding defined in rule $R3$ to estimate the preceding car's velocity. For example, if we look at the range of α from 0 to 0.12, we always get the same value in the term $\left[(1-\alpha)\cdot v_p + \frac{1}{2}\right] = \left[v_p + \frac{1}{2}\right]$

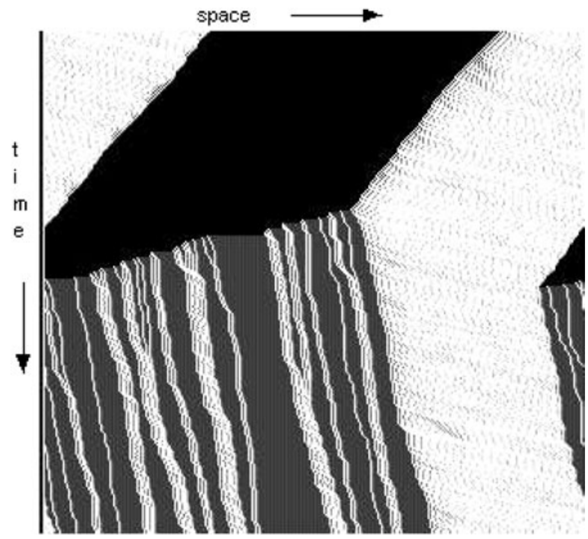

Figure 2. Space–time diagram for $R = 0.2$, $\rho = 0.5$ showing the time evolution of a system simulated initially with $\alpha = 0.51$. After switching to $\alpha = 0.5$, the behaviour changes dramatically.

(rule $R3$) for all values of v_p. So, for all data presented in the remaining part of this section we have chosen representative values for α: $\alpha = (0.12, 0.13, 0.2, 0.5, 0.9)$.

From figure 1 the impact of the driving strategies coded in α can be observed. Smaller values of α, i.e. higher anticipation levels, imply larger flows. Here vehicles maintain a less safe distance, leading to an increase in vehicular capacity. This behaviour is in agreement with, for example, platooning strategies that exploit knowledge of the velocity of preceding vehicles and require smaller headways (near 1 m), to increase the flow.

It is important to note that for values of α from 0.13 to 0.50 a second positive slope corresponding to a mixed branch is observed in the fundamental diagram. It is interesting that the initial positive slope, corresponding to a free-flow region where there are no slow vehicles, is similar for all values of α. Here the vehicles travel at near maximum speed. For the second branch, on the other hand, the flow is increased with non-maximum velocity, indicating a mixed region due to anticipation effects (figure 1 (right)). In order to analyse the role of the anticipation, we show the average velocity as a function of density for the same parameter values as in figure 1 (left).

As we can see from figure 1, higher levels of anticipation (smaller values of α) imply a larger density interval for the free-flow region. For values $0.13 < \alpha < 0.50$, after the free-flow region, traffic flow organizes in a so-called mixed region with a lower mean velocity. In this mixed region, in addition to free-flowing vehicles, vehicles moving in platoons where all cars have the same velocity and vanishing headway exist. The existence of this mixed region indicates that a suitable estimation of the velocity of preceding vehicles, coded in α, allows the flow to increase even for large densities.

Figure 2 shows a spacetime diagram[4] that exemplifies the dramatic changes in the microscopic structure when changing the value of α. Each horizontal row of dots represents the instantaneous positions of the vehicles moving towards the right, while successive rows of dots represent the position of the same vehicles at successive time-steps. The simulation is started with $\alpha = 0.51$ where the jamming regions travel backwards. However, after some

[4] For a Java applet of the simulations, see http://www.cie.unam.mx/xml/tc/ft/arp/simulation.html.

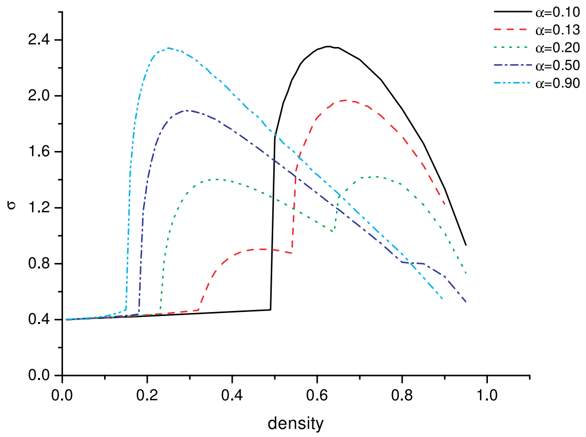

Figure 3. Standard deviation of the speed.

time we switch to $\alpha = 0.5$ and immediately observe a dramatic change in the slope of the congested regions. They are now travelling forward. This behaviour has been observed before in anticipatory modelling [18]. Such structured flow observed in spacetime diagrams increases the highway capacity due to the space reduction among vehicles. In these simulations we have found that the branches of congested or jammed flow collapse to a single region as in the VDR model [31]. These results are analogous to those for slow-to-start models, because effectively the outflow from a jam is reduced compared to the maximal flow.

On the other hand, it is also important to analyse the effects of different driving strategies on traffic safety. This can be done by an analysis of the standard deviation of vehicles' speed. A large standard deviation of speed means that, on average, a vehicle experiences frequent speed changes. In turn, the high speed variance could also increase the probability of traffic accidents. Integration of different driving strategies and traffic safety should be considered in order to facilitate more efficient road use, i.e. traffic with higher flow and minimal speed variance. From the thermodynamic point of view, the efficiency of a system is related to the entropy [27]. In particular in non-equilibrium situations, the entropy production is used to determine efficient conditions. In our case, a smaller standard deviation of speed indicates that the system is less disordered (in the velocity sense) since a system with less entropy production is more efficient than one with higher entropy production [28, 29]. In this way, the analysis of the standard deviation allows us to draw conclusions about safety and order in the system.

Figure 3 shows the standard deviation of speed resulting from our model for different strengths of anticipation. For each value of α a maximum that occurs shortly after the free-flow region can be clearly seen. In the free-flow region, the speed variance is negligible since there are no slow vehicles and fluctuations are extremely rare. Here we will use efficiency as an indicator of the higher flux in correspondence with order and safety conditions, i.e., high flux and small speed variance. Since the free-flow region increases as α decreases, it seems reasonable to attempt traffic with the higher level of anticipation in the range of density from 0 to 0.49. This choice not only produces a state with higher flow, but also the lowest standard deviation, so attaining more efficient traffic. Beyond this efficient density, the strength of anticipation coded in α should be changed based on the values of the standard deviation: for $\rho \in [0.5, 0.54)$, a more efficient performance is found with $\alpha = 0.13$; however, for $\rho \in [0.54, 0.63)$ more efficient behaviour is attained with $\alpha = 0.20$. Summarizing, the behaviour observed in figure 3 helps us to understand the necessity of a suitable use of different

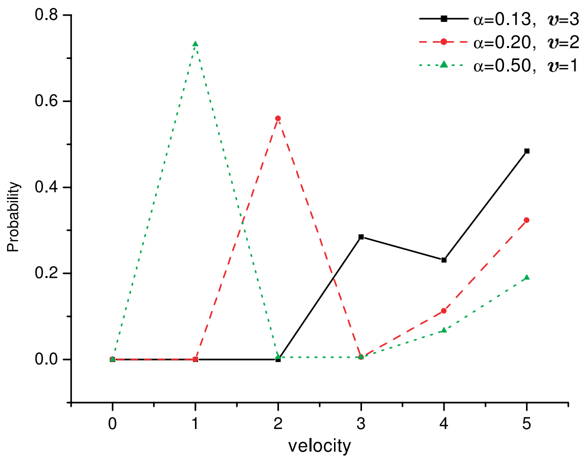

Figure 4. Velocity distributions for density $\rho = 0.4$ and different values of α. The v-values indicate the v-platoons found.

driving strategies to improve the efficiency of road use. In order to attain a compromise between traffic safety and capacity, the strength of anticipation should be determined depending on density: higher densities require a larger safety distance among cars, i.e. values of α closer to 1, in order to preserve safety levels.

In addition to homogeneous systems we have also investigated inhomogeneous ones where a random value of $\alpha \in [0, 1]$ is assigned to each vehicle [30]. This value is not changed during the time evolution to simulate a distribution of aggressive, non-aggressive and relaxed drivers. In this case we find that the region where the standard deviation is negligible is close to that resulting from homogeneous drivers with $\alpha = 0.2$ (high degree of anticipation). However, the variance of vehicle speeds is much larger than that corresponding to homogeneous drivers. In the fundamental diagram of the inhomogeneous system the mixed region is missing. Instead the variance in the level of anticipation produces higher fluctuations of speeds, and the flow decreases rapidly. Therefore some anticipation driving schemes have a strong impact on the behaviour of the system.

3.2. Structure of the mixed states

The behaviour in the mixed states is determined by the existence of dense platoons in which vehicles move coherently with the same velocity v. In the following these will be called v-platoons. The stationary state then shows phase separation into a free-flow region and a v-platoon. This is similar to the behaviour observed in models with slow-to-start rules where the system separates into free flow and a dense jam, i.e. a 0-platoon [31]. In our case, the behaviour of mixed states also exhibits phase separation into free flow and a condensed phase in which particles move coherently with finite velocity. To our knowledge, such behaviour has not been observed in other models before. It is also of great theoretical interest since recently general conditions for the occurrence of phase separation in driven diffusive systems have been suggested. The mixed state might be regarded as an idealization of homogeneous-in-speed states discussed in the context of synchronized traffic (see [32] and references therein).

Figure 4 shows the velocity distributions of the different branches. It can be clearly seen that only cars with velocity v and free-flowing cars (with velocity $v_{\max}$ or $v_{\max} - 1$ due to the randomization) exist. Due to the complexity of the model, an exact solution of the corresponding master equation cannot be derived. Such solutions so far have been found only in very special cases, i.e. models with $v_{\max} = 1$ [5, 6]. In the following, we present a phenomenological analysis based on the microscopic structure of mixed states leading to analytical expressions which describe with great accuracy the phase separation in the fundamental diagram. We start analysing the stability of such phases.

Since the headway d_i of a car i inside a v-platoon is $d_i = 0$, its new velocity is determined by $v_i' = \min\left(v_i, d_i^s\right)$ with $d_i^s = \left[(1-\alpha)\cdot v_p + \frac{1}{2}\right]$. For a stable v-platoon, v_i' must be equal to v. This gives the following stability condition:

$$v \leqslant (1-\alpha)\cdot v + \tfrac{1}{2}. \tag{1}$$

Equation (1) can be regarded as a condition for the anticipation parameter α. It implies that a v-platoon can only be stable for

$$\alpha_{v+1} < \alpha \leqslant \alpha_v \tag{2}$$

where α_v is defined by $\alpha_v := 1/2v$. However, this condition is only necessary, not sufficient. The v-platoons that can be realized for a given α also depend on the randomization R. For example, for $R = 0.2$, platoons with $v = 0, 1, 2, 3$ occur, whereas for $R = 0.4$ platoons with $v = 3$ cannot be observed in the infinite system although they might exist in small systems. Simulations indicate that the slope of the mixed branch in the fundamental diagram has to be smaller than $(1-R)(v_f - 1)$ where v_f is the average velocity in free flow: $v_f = v_{\max} - R$. This will be discussed in section 3.4 in more detail.

Another criterion for the stability of platoons can be derived from the condition that the inflow and outflow of the platoon have to be identical in the steady state. In the following we will derive estimates for these flows and in this way obtain analytical expressions for the fundamental diagram in the mixed region.

The outflow from a v-platoon is determined by the average time T_w needed by the leading vehicle of the platoon to accelerate to velocity $v+1$. Assuming that this car has a large headway, T_w is determined by the randomization constant R through $T_w = \frac{1}{1-R}$. Therefore, in the free-flow region of the system, the average headway Δx_f is $\Delta x_f = T_w(v_f - v) + 1$. This consideration is very similar to the reasoning used in [31].

Assuming that the platoon consists of N_v and the free-flow region of N_f vehicles, we have

$$N = N_v + N_f \qquad \text{and} \qquad L = N_v + N_f\Delta x_f. \tag{3}$$

Furthermore, it has been assumed that the transition region between the platoon, where all cars have headway $d_i = 0$, and the free-flow region, where the average headway is given by Δx_f, can be neglected. Eliminating N_f we find

$$\frac{N_v}{L} = \frac{\rho\Delta x_f - 1}{\Delta x_f - 1}. \tag{4}$$

We now can calculate the flow $J = \rho\bar{v}$ of the corresponding phase-separated state. The average velocity $\bar{v}$ in the presence of a v-platoon is given by

$$\bar{v} = \frac{N_v v + N_f v_f}{N}. \tag{5}$$

A straightforward calculation using the results given above yields for the flow in the mixed state

$$J_v = (1-R) + (v - (1-R))\rho. \tag{6}$$

These results are in excellent agreement with the results from computer simulations (see figure 5). Since $1 - R < 1$, all slopes corresponding to mixed states in the fundamental

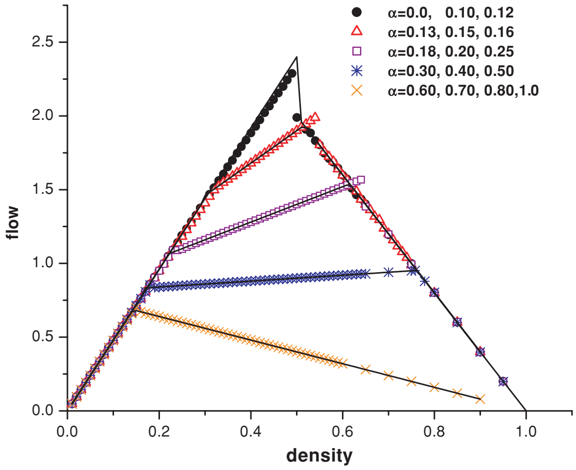

Figure 5. Comparison of the simulation results (symbols) with the analytical predictions (6) and (9) (solid lines).

diagram are positive, except those for 0-platoons which are responsible for the jammed branch with low flow and negative slope.

3.3. Structure of the congested states

One of the most interesting findings observed in the fundamental diagram is that all the different driving strategies attain the same congested curve, i.e., for large densities all curves collapse on one congested branch where the flow decreases with increasing density. In the following we explain this by analysing the structure of the congested states. Simulations indicate that the structure of the corresponding states depends on the parameter regime. In the range (2), where a v-platoon can exist, the congested branch is characterized by the coexistence of a compact jam (0-platoon) and various v-platoons. The v-platoons are formed when a bunch of vehicles escapes from the jam. As argued in section 3.2, the first car escapes after an average waiting time $T_w = \frac{1}{1-R}$. Due to anticipation, with probability $1 - R$ the second car can move in the same time-step, and so on. The average number of cars escaping in the same time-step is then given by

$$\bar{l} = \frac{\sum_{l=1}^{\infty} l(1-R)^l}{\sum_{l=1}^{\infty}(1-R)^l} = \frac{1}{R}. \tag{7}$$

These cars form a v-platoon of length $\bar{l}$ where the value of v depends on the parameter region as discussed above. Since the average waiting time for the escape of a car is T_w, the average distance between two v-platoons is $\Delta x_c = vT_w = \frac{v}{1-R}$.

To calculate the flow in the congested branch, we again neglect the transition regions and assume that only one jam with N_0 vehicles and n v-platoons with a total number of N_v cars are present. Then we have $N_0 + N_v = N$ with $N_v = n\bar{l}$. Furthermore $N_0 + N_v + n\Delta x_c = L$, where N_0 and N_v are the total lengths of the platoons and $n\Delta x_c$ is the total space between the platoons. These relations yield

$$1 = \frac{1}{L}(N + n\Delta x_c) = \rho + \frac{N_v}{L} \cdot \frac{\Delta x_c}{\bar{l}}. \tag{8}$$

The average velocity of the vehicles in the congested branch is $\bar{v} = \frac{N_v v}{N}$. Using (8), this implies for the flow

$$J_{\text{cong}} = \rho \bar{v} = \frac{N}{L}\frac{N_v}{N} v = (1-\rho)\frac{\bar{l}}{\Delta x_c} v = \frac{1-R}{R}(1-\rho). \tag{9}$$

Note that this result is independent of the velocity v of the platoons! It is in excellent agreement with the simulation data (see figure 5), justifying, e.g., the assumption made about the transition regions.

3.4. Stability regions

For fixed α we now can estimate the stability region $\rho_1 \leqslant \rho \leqslant \rho_2$ for the mixed states. At the lower boundary density ρ_1 the number of cars N_v in the v-platoon vanishes. From (4) one has $\rho_1 \Delta x_f - 1 = 0$ which yields

$$\rho_1 = \frac{1-R}{v_f - v + (1-R)}. \tag{10}$$

The upper bound ρ_2 is not determined by the condition $N_v = N$, i.e. all cars belong to the v-platoon. This would correspond to the density $\rho = 1$. In fact, the instability of the mixed state occurs earlier. At the density

$$\rho_2 = \frac{(1-R)^2}{R(v+R-2)+1} \tag{11}$$

the flow (6) of the mixed branch becomes larger than that of the congested branch (9) and therefore (at least for random initial conditions) the flow of the congested branch is observed. However, our simulations have given indications for hysteresis effects and metastability in the large density regime. We will discuss these in more detail in a future publication [30]. For $v = 0$ the upper transition density becomes $\rho_2 = 1$, independent of R, consistent with the observation (figure 1) that the mixed region for $v = 0$ extends up to the maximal density.

Since ρ_2 has to be larger than ρ_1, this yields an additional condition for the stability of the branches. It is easy to check that $\rho_1 < \rho_2$ if

$$(1-R)v_f > v. \tag{12}$$

This is just the condition obtained in section 3.2 from computer simulations.

Summarizing, a mixed region with v-platoons can only exist for $1/(2(v+1)) < \alpha \leqslant 1/(2v)$ and R satisfying (12). If these conditions are fulfilled, v-platoons occur in the density interval $\rho_1 \leqslant \rho \leqslant \rho_2$ where ρ_1 and ρ_2 are given by (10) and (11), respectively.

4. Summary and conclusions

Forecasting the impact of different anticipation schemes plays an essential role in real traffic flow in order to propose automated traffic alternatives. In this paper, we have introduced and investigated a modification of the NaSch model to better capture reactions of the drivers intended to maintain safety on the highway. The addition of an anticipation parameter $\alpha \in [0, 1]$ proves to be useful to describe different traffic situations of non-automated, automated and mixed traffic. Simulation results for driving schemes associated with intermediate levels of anticipation with α from 0.13 to 0.5, exhibit phase separation in a certain density regime $[\rho_1, \rho_2]$ (see (10), (11)) into a free-flow region and so-called

v-platoons. In these dense platoons vehicles move with the same velocity v and have a vanishing headway. The velocity v of the platoon is determined by the strength of anticipation and randomization through equations (2) and (12). These states are similar to the empirical observed homogeneous-in-speed states.

This platoon formation observed in a mixed regime plays an important role in automated highway systems in increasing the vehicular capacity. Therefore, the results obtained help to elucidate the effects of anticipation coded in α. Smaller values of α (greater estimation of the preceding car velocity) imply larger flows and a larger density interval for the free-flow regime. This is in accordance with, for example, the use of certain anticipation strategies to exploit knowledge of the velocity of the preceding vehicle and so reducing the distance among vehicles, increasing the capacity and the density interval for free-flow regime.

Moreover, the analysis of the speed variance of individual vehicles indicates the importance of establishing a suitable velocity anticipation scheme according to the density regime. The strength of anticipation should be determined based on the density in order to provide more efficient road use and so improving traffic safety. The highest strength of anticipation should be considered before the corresponding maximum density for the free-flow regime is reached. This choice not only produces traffic with maximum flow, but also with minimum speed variance. Higher densities require larger safe distances in order to preserve safety levels. Integration between capacity and safety resulting from the analysis of speed variance can help to improve the behaviour of traffic flow.

The considerations in this paper show the flexibility of the CA approach to more complex traffic flow problems. A simple and natural modification of the rules of the NaSch model to consider different driving schemes allows us to describe the formation of coherently moving platoons observed in some anticipation schemes. We think that the results presented here are relevant to establish suitable levels of safety and anticipation not only for AHS, but also in real traffic. We stress that although in this paper the model is simulated in a single lane on a ring, it is possible to apply it to complex highway topologies in a satisfactory way [30].

Apart from its practical relevance for traffic problems, our work also shows interesting physical aspects. The model suggested here exhibits various kinds of phase separation phenomena. At intermediate densities, phase separation into a condensed (v-platoon) and a non-condensed (free-flow) phase can be observed. In contrast to most other models of driven diffusion, the condensed phase moves coherently for $v > 0$. At high densities an even more surprising state is found that exhibits phase separation between different condensates, a non-moving one ($v = 0$) and several coherently moving platoons ($v > 0$). To our knowledge such behaviour has not been observed before. It would be interesting to study these phases in more detail, especially since recently some progress in the understanding of phase separation in driven diffusive models has been made [22, 23]. Work in this direction is currently in progress [30].

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